

MATHEMATICS SUBJECT KNOWLEDGE OF IRISH PRIMARY PRE-SERVICE TEACHERS

Dolores Corcoran, St Patrick's College, Drumcondra

Paper presented at the European Conference on Educational Research, University College Dublin,
7-10 September 2005

The purpose of this study was to establish what mathematics Irish primary school student teachers bring to initial teacher education. The research was premised on the belief that a certain kind of mathematics subject knowledge is needed for teaching and the possibility that such mathematics subject knowledge can be assessed. Three recently published curriculum implantation studies express concern that not all is well with the teaching of mathematics in Irish primary schools (Government of Ireland, 2005a, b; National Council for Curriculum and Assessment, 2005) These provide a background in praxis for the study into mathematics subject knowledge of primary pre-service teachers described here.

THE BACKGROUND

Shulman's (1986) attestation that "a century ago the defining characteristic of pedagogical accomplishment was knowledge of content" is confirmed by Oliver Goldsmith's description of the mathematical subject knowledge of an Irish schoolmaster in the eighteenth century which leaves the reader in no doubt about his mathematical abilities. The ensuing lines,

And still they gazed, and still the wonder grew

That one small head could carry all he knew (Goldsmith, c.1926)

indicate that he was very much more knowledgeable than his pupils or their parents. Contrast that with the picture, which the following passage from a contemporary Irish government report paints of mathematics teaching in Irish primary schools in the early twentieth century.

[revised Mathematics programme] a second objection was the difficulty experienced by the teachers in interpreting its significance and extent...Moreover, the present state of mathematical knowledge among women-teachers left us no alternative but to suggest that both Algebra and Geometry be optional for all women-teachers -- at least for some time to come until a considerable proportion of women-teachers are qualified to teach these subjects. For ensuring that all teachers may understand easily and exactly the meaning of the Programme, the best plan would be we suggest, that the Department issue detailed specimen-syllabuses.

(Dept of Education, 1926)

This gender-biased indictment of Irish primary mathematics teachers flags a dismal culture of mathematics curriculum implementation, which hinges on a deficit model

of teachers' subject knowledge in mathematics. Today's typical portrait of an Irish primary school teacher (Drudy, 2004) is of a relatively well paid, well educated woman struggling to teach a multiplicity of subjects from a centrally dictated curriculum to a class of up to thirty children whose interests, abilities and cultural capital vary enormously. This paper proposes to explore the mathematics subject knowledge of pre-service primary teachers, both substantive and syntactic (Grossman, Wilson and Shulman, 1989) with reference to that needed to teach the Primary Curriculum well (Gov. of I. 1999a, b). The image of educational research as a 'montage' is rehearsed by Denzin and Lincoln (2003). The findings make no claim to generalizability. They offer instead a series of 'snap shots' of the mathematics subject knowledge of the current cohort of student teachers in one college of education.

THE GROUP PHOTOGRAPH

St Patrick's College, in Dublin, has an annual intake in excess of 400 students on the Bachelor of Education (B Ed) programme. The great majority of these are recent school leavers who undertake a three-year degree course. A hundred others pursue a Post Graduate Diploma in Education (PG) and already hold diverse first degrees. Admission to Irish third level institutions¹ depends on a competitive system where the points awarded on six nominated leaving certificate subject results are aggregated. In the case of teacher education, mathematics must be included. Students are admitted to either B Ed or PGDE programme with a minimum of a D3 grade on a higher or ordinary level mathematics paper in the leaving certificate. These warrant 45 points and 5 points respectively out of a possible 100². In fact, almost all students

¹ Irish System of Schooling

Age of pupils	Type of schooling	Duration	Attendance	Examinations
4 - 12 years	Primary	8 years	Optional to age 6	-----
13-18 years	Second level	5 -6 years	Compulsory to age 15	Junior Certificate Leaving Certificate
18 + years	Third level	3 + years	Optional	Degree/diploma

² Irish Leaving Certificate Examination Points in Mathematics Calculation Grid

L. Cert. grade	Higher paper	Lower paper	L. Cert. grade	Higher paper	Lower paper
A1	100	60	C2	65	25
A2	90	50	C3	60	20
B1	85	45	D1	55	15
B2	80	40	D2	50	10

exceed this requirement. The great majority of B Ed students in the study took the leaving certificate in 2002 and entry requirements to the B Ed course that year were at a median of 480 points and a final cut off point of 455. A typical B Ed student therefore comes well within the top quintile of candidates' performance in the leaving certificate examination. There are some differences in education and experience between the two groups at course intake, but the disproportion of second year B Ed (N = 57) to PG (N= 32) students in this study makes it difficult to compare samples in a meaningful way. No distinction is made between teachers once they have been appointed to schools. As a consequence the two cohorts have been merged for purposes of data analysis and reporting.

THE AUDIT INSTRUMENT

The mathematics subject knowledge instrument used in this study was developed by a team of teacher educators for the SKIMA (Subject Knowledge in Mathematics) project in the UK, in response to a government directive mandating providers of initial teacher education to audit trainee teachers' subject knowledge in mathematics, and to remedy the deficiencies identified (Circular 4/98, DfEE, 1998). While not designed specifically for the Irish educational scene, the SKIMA audit gives an indication of some of the mathematical strengths and weaknesses of student teachers as they begin their mathematics teacher education (Rowland, Martyn, Barber and Heal, 2001). Analysis of the initial data set helps to outline a students' mathematics subject knowledge for teaching as they finish secondary school. This might be deemed an aspect of the raw material with which student teachers and mathematics teacher educators must work. The research reported here is part of a larger mixed methods (Burke Johnson and Onwuegbuzie, 2004) pilot study. To contribute detail to the picture of a student teacher's mathematics subject knowledge, each student rated his or her self-confidence in approaching each test item. Each student also indicated what mathematics he or she considered relevant to the primary school curriculum. (A detailed analysis of these findings will be presented elsewhere).

THE GENERAL PICTURE

There is considerable variation between the performances of Irish student teachers on the SKIMA self-audit. There is strong evidence that some students come well prepared mathematically to teach the primary school programme. But some students do badly. There are also marked differences between scores by the same students on different elements of the audit.

The sixteen SKIMA audit items can be grouped under the following headings: Number, Number Operations and Algebraic Thinking, Reasoning and Proof and

B3	75	35	D3	45	5
C1	70	30			

Measures, Shape and Space. These can be loosely mapped with varying degrees of match to the five strands of the Irish mathematics curriculum, which are: Number, Algebra, Measures, Shape and Space and Data. Irish student teachers are generally strong on number and operations on number, but a distinction can be made here between operative mathematical understanding and descriptive understanding (Sierpinkska, in Watson and Mason, 1998). The problematic nature of question five which occurs in the group of questions designed to test Number Operations and Algebraic Thinking involves a straightforward multi-digit multiplication calculation which almost all students performed correctly but it also required knowledge of the application of the associative property of multiplication and this piece of syntactic mathematical subject knowledge appears to have escaped 93.3% of students.

Table 1: Some quantitative data concerning all participants' performance in each of the sixteen questions.³

Audit item	Mean score	Standard deviation	Mode	Score of 3-4 'secure response'	Score of 0-1 'insecure'
Q 1	3.58	1.42	4	86.5%	1.1%
Q 2	3.62	.94	4	89.8%	2.2%
Q 3	3.92	.40	4	97.7%	1.1%
Q 4	2.30	1.52	4	48.3%	16.9%
Q 5	2.02	.56	2	6.7%	1.1%
Q 6	3.19	1.12	4	66.3%	1.1%
Q 7	2.78	1.44	4	59.5%	6.7%
Q 8	2.46	1.70	4	56.1%	22.5%
Q 9	1.83	1.47	1	30.3%	19.1%
Q 10	2.82	1.52	4	60.6%	12.4%
Q 11	1.42	1.31	1	21.4%	24.7%
Q 12	1.44	1.23	1	19.1%	25.8%

³ Marking Scheme for SKIMA Maths Subject Knowledge Audit; Marks: 0, 1, 2, 3, 4.

0 – not attempted, no progress towards a solution

1 – insecure- partial solution, incorrect

2 – secure in parts, insecure in parts

3 – secure, small errors, explanations acceptable but not completely convincing

4 – completely secure with convincing and rigorous explanations

Q 13	2.59	1.20	4	53.9%	4.5%
Q 14	1.05	1.11	1	9%	33.7%
Q 15	1.82	1.40	1	31.4%	15.7%
Q 16	.26	.63	0	1.1%	82%

Irish student teachers did least well on the group of questions that seek to assess understanding of Shape, Space and Measures. Question 16 involves knowledge of transformations and at a concrete level, teachers of the primary mathematics curriculum would need some knowledge of symmetry, which it seeks to assess. The indications are that Geometry as taught in second level schools and assessed in the leaving certificate does not prepare students for this question. Findings from PISA 2003 corroborate that this area of ‘mathematical literacy’ is the domain in which Irish students score least well (Cosgrave, Shiel, Sofroniou, Zastrutzki and Shortt, 2005). Question 14, an item requiring that they find the area and perimeter of a parallelogram set against a squared grid also appears highly problematic for Irish student teachers with only 9% of the cohort providing a ‘secure response’.

Table 2: Audit topics with the highest and lowest facility ratings

Highest facility		Lowest facility	
% secure	Topic	% secure	Topic
[Q3] 97.7%	Equivalent values for fractions, decimals & percent	[Q16] 7.8%	Geometrical Transformations
[Q2] 89.8%	Ordering fractions	[Q14] 9.0%	Perimeter and area of a parallelogram
[Q1] 86.5%	Ordering decimals	[Q12] 19.1%	Reasoning and proof in the context of perimeter and area
[Q6] 66.3%	Long division	[Q11] 21.4%	Function, equation and graph in context of paying a plumber

Findings of the 1999 National Assessment of Mathematics Achievement (Shiel and Kelly, 2001) indicate that the test items in which students in fourth class in primary schools performed least well were those requiring higher level mathematical processes such as mathematical reasoning (55%), analysing and solving problems and evaluating solutions (54%) and understanding and making connections between mathematical processes and concepts (53%). Of the six mathematical process skills listed in the Primary Mathematics Curriculum (Gov. of I. 1999a), development of applying and problem solving skills, together with formal reasoning, mark this curriculum as new and different. Their development in children requires considerable depth of mathematics subject knowledge on the part of teachers in order to invent or

select problems suitable for teaching a particular mathematics topic to particular children and personal confidence in one's own ability to reason mathematically in order to structure lessons which will develop children's reasoning skills. Such depth of subject knowledge appears lacking in all but 20% of these Irish student teachers as evidenced by answers to item 12 below.

SNAPSHOTS OF REASONING AND POOF

U K Circular 4/ 98 requires that trainee teachers demonstrate:

That they know and understand {...} methods of proof, including simple deductive proof, proof by exhaustion and disproof by counter-example (DfEE, 1988, p. 62)

No such knowledge is explicitly required of Irish primary teachers but the "crucial role" of the teacher "in guiding the child to construct meaning, to develop mathematical strategies for solving problems and to develop self-motivation in mathematical activity" (Gov of I, 1999a:5) is highlighted. Explicitly, the primary curriculum requires that children develop the skill of **Reasoning**. This is elaborated that *the child should be enabled to:*

- make hypotheses and carry out experiments to test them
- make informal deductions
- search for and investigate mathematical patterns and relationships
- reason systematically in a mathematical context
- justify processes and results of mathematical activities, problems and projects.

Figure 1: Item 12

120 square tiles can be made into a rectangular mosaic. The sides of each tile are 1 cm. The shape of the rectangle can vary. For example, it might be 10 tiles by 12 tiles.

State whether each of the following three statements is true or false.

Justify your claims in an appropriate way.

- (a) The perimeter (in cm) of every such rectangle is an even number
- (b) The perimeter (in cm) of every such rectangle is a multiple of 4
- (c) No such rectangle is a square

SKIMA self-audit item Q 12 might be said to assess student teachers' facility with mathematical reasoning. Fewer than 20% of students made a secure response to the whole question. Less than 25% were 'secure in parts/insecure in parts'. A further 31.5% scored one mark, while 25.8% made no response or offered 'incorrect answers to two or more parts – no justification or justification only applying to the 10 x 12 case'. In an effort to understand the thinking strategies of Irish student teachers the detailed responses of three students are examined in depth. The students were selected at random from the data set.

Close-up One

Aoife's raw score of 48 translated to a total of 75% on the SKIMA self-audit. Earlier, she had achieved grade A on the ordinary level mathematics paper at leaving certificate (60 points) and might be said to have done well overall on the self-audit. For those questions where she rated her confidence, she appears to have an accurate picture of the mathematics she can do, but her assessment of curricular relevance confirms an interpretation of the primary mathematics curriculum as being primarily about number concepts and operations on numbers.

Table 3: Aoife's scoring profile on the self-audit:⁴

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16
5	---	5	---	5	5	3	5	5	---	----	3	5	3	----	2
4	4	4	4	2	4	4	3	2	1	1	1	4	2	2	4
3	2	4	2	4	4	2	2	2	0	1	2	1	2	1	1

Aoife selects *a* as 'true' and justifies her answer in the following manner:

$$\text{Perimeter} = 2 \text{ lengths} + 2 \text{ breadths}$$

When you multiply 2 odd nos. you get an even number e.g. $2 \times 3 = 6$

$$P = 2(l) + 2(b) \quad 2(9) + 2(13) \quad \text{or} \quad 2(7) + 2(15)$$

$$18 + 26 = 44 \qquad 14 + 30 = 44$$

She then selects *b* as true and justifies (incorrectly) as follows:

Perimeter will always remain the same as a rectangle will always take up the same amount of space despite changing its width and length.

$$\text{e.g. } P = 2(l) + 2(b) \qquad \text{or } B = 5, L = 17$$

B, L

⁴ Coding key: Row 2 indicates her confidence level in attempting the item. Row 3 her actual score for the item. Row 4 indicates her opinion as to the relevance to the primary curriculum of the mathematics accessed by the audit item.

$$\begin{aligned}
8, 14 &= 2(8) + 2(14) \\
&= 16 + 28 \\
&= 44
\end{aligned}$$

$$\begin{aligned}
2(5) + 2(17) \\
10 + 34 \\
= 44
\end{aligned}$$

She incorrectly indicates part *c* is false and justifies this by:

You are using 22 tiles all the time therefore it is possible to create a square
11 x 11

It appears that Aoife ignored the opening statement of this problem and read ‘any such rectangle’ to mean *any* rectangle. It is difficult to conjecture why she chose the example of length and breadth of a rectangle as 9 x 13, except as a possible pair of odd numbers, and the next example follows in that she chose the next two consecutive odd numbers respectively. In part *b*, she continues to use 44 as an example of perimeter but appears to confuse the conservation of area with perimeter. Rowland *et al* (2001) found this same ‘false conservation misconception’ (Lunzer, 1968) among student teachers in England. It is difficult to understand where Aoife got 22 tiles “all the time” in part *c*, unless she thought of it as half the previous perimeter and so some combination of 22 would equal length plus breadth. Obviously she presumed the perimeter was fixed at either 44 or 22. The suggestion that it is possible to create an 11 x11 square with 22 tiles indicates a poor number sense out of line with her perceived curricular priorities.

Close-up Two

Ciara scored B3 on an honours leaving certificate mathematics paper in the leaving certificate (worth 75 points) and has studied extra mathematics courses since. Her raw score on the SKIMA audit was 52, which translated to 81.25%.

Table 4: Ciara’s scoring profile on the self-audit:⁵

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16
5	4	5	5	4	5	3	5	5	---	5	4	3	3	5	3
4	4	4	4	2	4	4	4	2	4	4	2	4	1	4	2
4	0	4	0	3	3	0	0	0	3	4	0	0	2	1	0

Ciara shows secure mathematical subject knowledge, except in relation to Q 12 and the use of the associative property of multiplication over addition (Q5), generalisation from a sequence of three consecutive numbers (Q9), correctly calculating the area and perimeter of a parallelogram (Q14) and the transformations question (Q16). A detailed analysis of question 12 reveals an interesting insight into her thinking. She starts by nominating two cases of such a rectangle by writing:

⁵ As 3

120 'square' (sic) = 10×12

3×40

Parts *a*, *b* and *c* are all deemed to be true and the same (incorrect) explanation is given for parts *a* and *b*...

a The perimeter of all rectangles will be 120, which is even,

b The perimeter of all rectangles will be 120, which is divisible by 4.

Part *c* is explained as

"True, as $10 \times 10 = 100 \neq 120$

and

$11 \times 11 = 121 \neq 120$

Therefore no real number squared can have a value of 120.

Part *c* is of course correct but parts *a* and *b* indicate a flawed understanding of the relationship between area and perimeter. In neither case is a proof offered to justify the claims made.

Close-up Three

Deirdre achieved B in ordinary level mathematics at leaving certificate (worth 40 points) and 56.25% overall on the self-audit.

Table 5: Deirdre's scoring profile on the self-audit:⁶

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16
5	5	5	3	5	5	3	3	5	3	---	3	3	5	--	--
4	4	4	4	2	4	4	2	1	2	---	2	2	0	2	0
4	4	4	1	4	4	3	2	4	0	2	1	1	4	0	0

Her statements in answer to all three parts of the question were correct but she only offered justification for part *b*. She drew an irregularly shaped 'square' and labelled each side 1 cm. This prompts the surmise that she was thinking in spatial terms about the composition of 'every such rectangle'. She then wrote

" $120/3 = 40$ " followed by "9 x" and then

40

⁶ As 3

arranged as the dimensions of a rectangle, which was not actually drawn. Beside this is $86/4$ with an answer of 21 (Though strictly incorrect, or unfinished, its enough to tell her that 86 is not divisible by 4). We can deduce from these symbols that Deirdre sought an example of the mosaic other than the 10×12 given in the question. Since its perimeter 86 is not divisible by 4 she has the beginnings a proof by counter example that the perimeter of every such rectangle is not divisible by four. Yet nothing at all is offered to justify the statement that no such rectangle is a square.

These three students all demonstrate a poor facility for mathematical reasoning and proof which has been found to be notoriously difficult to develop over a short mathematics education course (Rowland *et al*, 2003, Goulding, 2003). Current research findings in Ireland indicate that self-styled ‘good’ successful teachers of mathematics at second level equate improved learning with the memorisation of formulae and procedures. Regrettably, improved learning of mathematics is “not equated with thinking creatively, being able to provide reasons for solutions, or understanding how mathematics is used in the real world” (Lyons, Lynch, Close, Sheerin and Boland, 2003, p. 366). The performance of students on SKIMA reflects this limited emphasis on the syntactical aspects of mathematics by their teachers.

THESE STUDENTS AS TEACHERS

Currently students of St Patrick’s College are graded on a scale from A to F as generalist primary school teachers for Teaching Practice (TP) in the following three key areas: the quality of planning and preparation; the quality of teaching and learning and the quality of professionalism.⁷ During spring teaching practice second year students prepare schemes for all subject areas and a minimum of four lesson plans per day. Students teach for three weeks and it is an expectation that they would

7

Teaching Practice Marking Scheme					
Grade	Percentage	Result	Grade	Percentage	Result
A +	80%	First class hon.	C +	55%	2.2 honours
A	75%	First class hon.	C	52%	Pass
A -	70%	First class hon.	C -	49%	Pass
B +	67%	2.1 honours	D +	45%	Pass
B	63%	2.1 honours	D	40%	Pass
B -	59%	2.2 honours	F		Fail

teach one of the three core subjects Irish, English or Mathematics every day. In practice, students commonly address each of these subjects during a different week of teaching practice. Each student is visited by his or her supervisor once a week. For these reasons students may not have had a mathematics lesson supervised, or may have been supervised by someone whose curricular interests lie elsewhere.

Student scores were distributed in a three by three matrix, showing mathematics subject knowledge as measured on the SKIMA self-audit on the *vertical axis* and TP scores for overall teaching performance on the *horizontal-axis*. Students were grouped as High, Middle and Low according as their SKIMA score fell between 64-100%, 51- 63%, and 28- 49 % respectively. TP scores were designated as ‘Strong’, ‘Capable’ or ‘Weak’ according as they fell between A and B, B- and C, and C- and F. F represents ‘failure’ and means a student has to repeat the TP session.

Table 6: Teaching Practice scores and SKIMA scores of the cohort (spring 2005)

SKIMA total scores per category		Second Year Spring Teaching Practice Scores		
SKIMA Mathematics self-audit scores	High 35%	Strong	Capable	Weak
		18%	13%	4%
	Middle 30%	Strong	Capable	Weak
		20%	10%	0%
	Low 34%	Strong	Capable	Weak
		18%	10%	4%

Indications from this table are that Irish student teachers do well in the classroom with 90% of them deemed capable or better in the teaching role. In consequence, we should be concerned about the 28% of students who are deemed ‘strong’ or ‘capable’ as teachers but exhibit a worrying lack of mathematics subject knowledge, which may translate into inadequate or poor teaching of mathematics. It may be argued that not all of the mathematics assessed by the SKIMA audit is essential to the work of Irish primary teachers. Yet almost all of it can be found among the teaching objectives of the mathematics curriculum (Gov of I, 1999 a, b). Darling Hammond’s assertion that “It makes sense that knowledge of the material to be taught is essential to good teaching, but also that returns to subject matter expertise would grow smaller beyond some minimal essential level which exceeds the demands of the curriculum being taught (2000, p.3)” may be true and Irish primary teachers don’t need the mathematics assessed in SKIMA item 16. However, because the TP grades used in this study were not focused explicitly on mathematics, we cannot assume that the “good teaching” accolade awarded to these students necessarily means “good mathematics teaching”. Close (1999, in Government of Ireland, 2002) contends that those student teachers who perform least well in mathematics in the leaving certificate tend to focus on mathematics as procedural rather than conceptual knowledge.

The distribution of the TP scores obtained by the students in this study against SKIMA audit scores over a contingency table does not agree with the earlier finding by Rowland *et al* (2001) that a significant association exists between audit score and teaching performance. The table has a $df = 4$ and values of chi-square less than 9.5 support the null hypothesis. For this table, Pearson chi – square value is 3.138, which upholds the null hypothesis that teaching performance and audit performance are independent of each other in this study. This finding is not unexpected given the general nature of the TP assessment. Perhaps if mathematics teaching were to become a focus of Teaching Practice assessment such a claim would no longer be true in the Irish context.

CONCLUSIONS

Because of the requirement to audit mathematical subject knowledge, English students would perhaps have been more prepared for the SKIMA audit than their Irish counterparts. In Ireland there is no obligation on students to indicate their proficiency in mathematics beyond the entry requirement.

Clear links can be made between mathematics subject knowledge assessed by SKIMA and that required to teach the primary mathematics curriculum well. Syntactic knowledge of generalisation and justification together with a deeper substantive knowledge of elementary shape and space concepts, functional thinking and measures are some of the minimal essential level of mathematics required to teach such a curriculum with the problem based approach recommended. Numerous references are to be found in the literature about the difficulty for teachers to teach mathematics in a manner, which is qualitatively different from that in which they themselves learned mathematics (Schifter and Bastable, 1995). Indications are that this difficulty remains for students particularly in a culture which values and develops generic teaching skills and competencies without due regard to the curricular requirement to develop mathematics process skills. The mathematics, which many student teachers bring to teacher education, is highly procedural and their mathematics knowledge base often lacks important syntactic elements. A shaky (or too rigid!) foundation may hamper further development of mathematics knowledge during a teaching career (Grossman, Wilson and Shulman, 1989).

Remediation sessions with an emphasis on process skills could help students reconceptualise mathematics as a creative activity, “an experimental test-bed, in which they might confidently respond to an unexpected student question ‘I don’t know let’s find out’” (Rowland *et al*, 2001, p. 125). The time constraints on current education courses dictate that such remediation sessions should involve peer tutoring (Barber, Heal and Martyn, 2003) outside of scheduled courses. Working mathematically in collaborative groups would a fruitful way for students to experience first hand the communicating and expressing skills of the mathematics curriculum. Ideally, the B Ed programme might be extended to a fourth year with a pro rata lengthening of the P G programme and could include a new course aimed

specifically at exploring and developing the pre-service teachers own mathematics subject knowledge so that they might become better teachers of mathematics.

References

- Barber, P. and Heal, P. (2003) Primary teacher trainees' mathematical subject knowledge: the effectiveness of peer tutoring. In J Williams (Ed.) *Proceedings of the British Society for Research into Learning Mathematics*, volume 23, number 3
- Burke Johnson, R. and Onwuegbuzie, A. (2004) Mixed Methods Research: A Research Paradigm Whose Time has Come. *Educational Researcher*, Vol. 33, No 7, pp.14-26
- Close, S. (1999) Pre-service Mathematics Education in St Patrick's College. Unpublished manuscript
- Cosgrave, J. Shiel, G. Sofroniou, N. Zastrutzki, S. and Shortt, F. (2005) *Education for Life: The Achievements of 15-Year Olds in Ireland in the second Cycle of PISA*. Dublin: Educational Research Centre
- Darling Hammond, L. (2000) Teacher Quality and Student Achievement: A Review of State Policy Evidence. *Education Policy Analysis Archives*. volume 8, no. 1, Online at <http://epaa.asu.edu>
- Denzin, N. and Lincoln, Y. (Eds) *Strategies of Qualitative Enquiry*. California: Sage Publications
- Department for Education and Employment (1998) *Teaching: High Status, High Standards*. Circular 4/98. London: DfEE
- Department of Education (1926) *Report and Programme; National Programme Conference*. Dublin: Government Publications
- Drudy, S. (2004) Gender and Initial Teacher Education in a Changing Context, paper presented at Educational Studies of Ireland Annual Conference. NUI, Maynooth, April
- Goldsmith, Oliver (c.1926) *The Deserted Village. At the Sign of the Three Candles*: Dublin
- Government of Ireland (1999a) *Primary School Mathematics Curriculum*. Dublin: Government Publications
- Government of Ireland (1999b) *Primary School Mathematics Teacher Guidelines*. Dublin: Government Publications
- Government of Ireland (2002) *Preparing Teachers for the 21st Century, Report of the Working Group on Primary Pre-Service Teacher Education*. Dublin: The Stationery Office

- Government of Ireland (2005a) *An Evaluation of Curriculum Implementation in Primary Schools*. Dublin: The Stationery Office
- Government of Ireland (2005b) *Literacy and Numeracy in Disadvantaged Schools: Challenges for Teachers and Learners*. Dublin: The Stationery Office
- Goulding, M. (2003) 'An investigation into the mathematical knowledge of primary trainees,' in J. Williams (Ed.) *Proceedings of the British Society for Research into Learning Mathematics*. volume 23, number 3
- Grossman, P. Wilson, S. and Shulman, L. (1989) Teachers of substance: subject matter knowledge for teaching, in M. Reynolds (Ed.) *Knowledge base for the beginning teacher*. Oxford: Pergamon
- Lyons, M. Lynch, K. Close. S. Sheerin. E. and Boland. P. (2003) *Inside Classrooms: The Teaching and Learning of Mathematics in Social Context*. Dublin: Institute of Education
- National Council for Curriculum and Assessment (2005) *Primary Curriculum Review, Phase One: Final Report*. Dublin: Author
- Rowland, T. Martyn, S. Barber, P. and Heal, C. (2001) 'Investigating the mathematics subject matter knowledge of pre-service elementary school teachers'. In M. van den Heuvel-Panhuizen (Ed.) *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education, Volume 4* pp. 121-128. Utrecht, The Netherlands.
- Rowland, T., Barber, P., Heal, C. and Martyn, S. (2003) 'Prospective primary teachers' mathematics subject knowledge: substance and consequence.' In J. Williams (Ed.) *Proceedings of the British Society for Research into Learning Mathematics*, volume 23, number 3, pp. 91-96
- Schifter, D., and Bastable, V., (1995) From the teachers' seminar to the classroom: The relationship between doing and teaching mathematics, an example from fractions. Paper presented at the American Educational Research Association, San Francisco, April
- Shiel, G. and Kelly, D. (2001) *The 1999 National Assessment of Mathematics Achievement*. Dublin: Educational Research Centre
- Shulman, L. (1986) 'Those Who Understand: Knowledge Growth in Teaching.' *Educational Researcher*, February
- Watson, A and Mason J. (1998) *Questions and Prompts for Mathematical Thinking*. Derby: Association of Teachers of Mathematics